MATH 239 Winter 2016: Assignment 3

Due: 10:00 AM, Friday January 29, 2016 in the dropboxes outside MC 4066

Last Name: First Name:

I.D. Number: Section:

Mark (For the marker only): /27

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1. For each of the following, determine the generating series of the set with respect to the weight function, and express the series as a simplified rational expression.

   (a) {3 marks} Set: $\mathbb{N}_0 = \{0, 1, 2, 3, \ldots\}$. Weight function: $w(a) = \begin{cases} a+1 & a \equiv 0 \pmod 3 \\ a & a \equiv 1 \pmod 3 \\ 2a & a \equiv 2 \pmod 3 \end{cases}$.

   (b) {3 marks} Set: $\{1, 2\} \times \{1, \ldots, 239\} \times \mathbb{N}_0$. Weight function: $w(a, b, c) = 3a + b + 4c$.

2. {5 marks} Let $m \in \mathbb{N}_0$. Suppose we have an unlimited supply of nickels, dimes and quarters (they are worth $5, 10, 25$ cents each, respectively). How many ways can we make m cents using these coins such that there are more quarters than dimes? Your solution must include the definition of a set and a weight function, and finding the relevant generating series. Your final answer may be expressed as the coefficient of a certain power series.

3. {8 marks} For any integer $n \geq 0$, determine the number of compositions of n with exactly 3 parts where exactly 2 of these parts are even. You need to define a relevant set, a weight function, determine a generating series, and then find an explicit formula for the answer. (Hint: Consider different cases.)

4. Let $q \geq 2$. For any $n \geq 0$, let A_n be the set of all compositions of n where every part is at least q, and let B_n be the set of all compositions of n where every part is either 1 or q. For example, when $q = 3$, the compositions $(3, 5, 6, 3, 3)$ and $(10, 10)$ are in A_{20}, while $(3, 1, 1, 3, 1, 1, 1, 3, 3, 3)$ and $(3, 3, 3, 3, 3, 3, 1, 1)$ are in B_{20}. (Note that the number of parts is not restricted.)

(a) {6 marks} Using generating series, determine $|A_n|$ and $|B_n|$ as coefficients of some simplified power series.

(b) {2 marks} Use your results in part (a) to prove that for all $n \geq 1$, $|A_n| = |B_n| - |B_{n-1}|$.